

NAG Toolbox for MATLAB

f08gc

1 Purpose

f08gc computes all the eigenvalues and, optionally, all the eigenvectors of a real symmetric matrix held in packed storage. If the eigenvectors are requested, then it uses a divide-and-conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal–Walker–Kahan variant of the QL or QR algorithm.

2 Syntax

```
[ap, w, z, info] = f08gc(job, uplo, n, ap)
```

3 Description

f08gc computes all the eigenvalues and, optionally, all the eigenvectors of a real symmetric matrix A (held in packed storage). In other words, it can compute the spectral factorization of A as

$$A = Z\Lambda Z^T,$$

where Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i , and Z is the orthogonal matrix whose columns are the eigenvectors z_i . Thus

$$Az_i = \lambda_i z_i, \quad i = 1, 2, \dots, n.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job** – string

Indicates whether eigenvectors are computed.

job = 'N'

Only eigenvalues are computed.

job = 'V'

Eigenvalues and eigenvectors are computed.

Constraint: **job** = 'N' or 'V'.

2: **uplo** – string

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'

The upper triangular part of A is stored.

uplo = 'L'

The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **n** – **int32 scalar**

n , the order of the matrix A .

Constraint: $n \geq 0$.

4: **ap**(*) – **double array**

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n symmetric matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;

if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

ldz, work, lwork, iwork, liwork

5.4 Output Parameters

1: **ap**(*) – **double array**

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

ap contains the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A .

2: **w**(*) – **double array**

Note: the dimension of the array **w** must be at least $\max(1, n)$.

The eigenvalues of the matrix A in ascending order.

3: **z**(ldz,*) – **double array**

The first dimension, **ldz**, of the array **z** must satisfy

if **job** = 'V', **ldz** $\geq \max(1, n)$;
if **job** = 'N', **ldz** ≥ 1 .

The second dimension of the array must be at least $\max(1, n)$ if **job** = 'V' and at least 1 if **job** = 'N'

If **job** = 'V', **z** contains the orthogonal matrix Z which contains the eigenvectors of A .

If **job** = 'N', **z** is not referenced.

4: **info** – **int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **uplo**, 3: **n**, 4: **ap**, 5: **w**, 6: **z**, 7: **ldz**, 8: **work**, 9: **lwork**, 10: **iwork**, 11: **liwork**, 12: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

if **info** = i and **job** = 'N', the algorithm failed to converge; i elements of an intermediate tridiagonal form did not converge to zero; if **info** = i and **job** = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and column $i/(n+1)$ through $\text{mod}(i, n+1)$.

7 Accuracy

The computed eigenvalues and eigenvectors are exact for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. See Section 4.7 of Anderson *et al.* 1999 for further details.

8 Further Comments

The complex analogue of this function is f08gq.

9 Example

```
job = 'V';
uplo = 'L';
n = int32(4);
ap = [1;
      2;
      3;
      4;
      2;
      3;
      4;
      3;
      4;
      3;
      4;
      4];
[apOut, w, z, info] = f08gc(job, uplo, n, ap)

apOut =
    1.0000
   -5.3852
    0.4062
    0.5416
   10.1724
   -1.8302
    0.7276
   -0.8160
    0.1223
   -0.3564
w =
   -2.0531
   -0.5146
```

```
-0.2943
12.8621
z =
-0.7003   -0.5144   -0.2767   -0.4103
-0.3592    0.4851    0.6634   -0.4422
 0.1569    0.5420   -0.6504   -0.5085
 0.5965   -0.4543    0.2457   -0.6144
info =
      0
```
